Research Contributions of Prof. O. S. K. S. Sastri from 5th September 2023 to 15th January 2025
Best Researcher Award

Research Publications:

1. Sharma, A., Awasthi, A., & **Sastri, O. S. K. S.** (2024). Novel approach to compute inverse potentials for charged particle systems applied to pp-scattering. *Nature Scientific Reports*. [Accepted for publication]. **[Q1]**
2. **Sastri, O. S. K. S.**, Sharma, A., & Awasthi, A. (2024). Constructing inverse scattering potentials for charged particles using a reference potential approach. ***Physical Review C***, *109*(6), 064004. **[Q1]**
3. Awasthi, A., Sharma, A., Kant, I., & **Sastri, O. S. K. S.** (2024). High-precision inverse potentials for neutron-proton scattering using piece-wise smooth Morse functions. ***Chinese Physics C***, *48*(10), 104104. **[Q1]**
4. Awasthi, A., & **Sastri, O. S. K. S.** (2024). Comparative study of α-α interaction potentials constructed using various phenomenological models. ***Turkish Journal of Physics***, *48*(3), 102-114. **[Q3]**
5. Awasthi, S., Kant, I., Khachi, A., & **Sastri, O. S. K. S.** (2024). Elastic scattering study of α-3 H and α-3 He modeled by Malfliet–Tjon potential using phase function method. ***Indian Journal of Physics***, 1-9. **[Q3]**
6. Khachi, A., Awasthi, S., Kumar, L., & **Sastri, O. S. K. S**. (2024). Algorithm to Obtain Inverse Potentials for α–α Scattering Using Variable Phase Approach. ***Computational Mathematics and Mathematical Physics***, *64*(10), 2320-2332. **[Q3]**
7. Sharma, A., Sharma, A., **& Sastri, O. S. K. S**. (2024). Simulation Study of Arbitrary 1D Periodic Potentials by Modified Marsiglio's Matrix Approach using Gnumeric Spreadsheet. ***Resonance***, *29*(12), 1627-1641. **[Q3]**
8. Awasthi, S., & **Sastri, O. S. K. S.** (2024). Real and imaginary phase shifts for nucleon–deuteron scattering using phase function method. ***Physics of Atomic Nuclei***, *87*(3), 311-318. **[Q4]**

Publications of UG and PG students' research work:

Following the directives of the Honorable Vice-Chancellor, publication credits have been formally integrated into the undergraduate and postgraduate programs to foster academic excellence and encourage scholarly contributions. To implement this initiative, all research projects undertaken by students have been meticulously prepared and successfully published in esteemed national and international conferences. This step not only enhances the academic profile of the institution but also provides students with invaluable exposure to global research standards and practices.

Poster Presented by PG students in 67th DAE Symposium on Nuclear Physics 2023 at IIT Indore.

1. **Yogesh, Simran** -Alpha preformation probability from Cluster Formation Model using AME-2020.
2. **Jyoti, Barbie-**Viola-Seaborg Coefficients for Partial Alpha Half-lives Based on Latest NNDC Data.

Poster Presented by PG students in International Conference on Physics for Sustainable Development (ICPSD-2024) at Central University of Jammu.

1. **Barbie:** Study of alpha - 12 Carbon Elastic Scattering using Phase Function Method.
2. **Simran:** Study of p-12 Carbon elastic scattering at astrophysical energies using phase function method.
3. **Jyoti:** Phase Shift Analysis of Deutron-Alpha Elastic Scattering 2+ Resonance.
4. **Yogesh:** Phase shift analysis of p-16 O elastic scattering at astrophysical energies using phase function method.

Poster Presented by BSc. Research Students in XXVI DAE-BRNS High Energy Physics Symposium 2024 at IIT Banaras Hindu University, Varanasi

1. **Tanisha:** Energy-Dependent Inverse Potentials for Neutron-Proton Elastic Scattering at energy up to 1050 MeV Using a Piecewise Morse Function.
2. **Gargi:** Optical Inverse potential for Prtoton -Protn up to 1GeV.
3. **Sanyam:** Isospin-Dependent Inverse Potentials for Elastic Pion-Nucleon Scattering in the S11 and S31 Channels.

Conference Proceedings:

1. Awasthi, S., & **Sastri, O. S. K. S**. (2023). Numerical simulation study of light nuclei. In *Proceedings of the DAE Symp. on Nucl. Phys.* (Vol. 67, p. 1435).
2. Awasthi, S., Khachi, A., & **Sastri, O. S. K. S.** (2022, December). Cross Section for 3 H (α, γ) 7 Li Astrophysical Reaction Using Scattering Phase Shifts. In *High Energy Physics Symposium* (pp. 983-984). Singapore: Springer Nature Singapore.

3. Awasthi, S., Kant, I., Khachi, A., & **Sastri, O. S. K. S**. (2023). Phase wave analysis of nucleon-α scattering using Malfliet-Tjon potential. In *Proceedings of the DAE Symp. on Nucl. Phys.* (Vol. 67, pp. 425–426).
4. Sharma, A., & **Sastri, O. S. K. S**. (2023). Modelling long range Coulomb interaction in pp-scattering studies using reference potential approach. In *Proceedings of the DAE Symp. on Nucl. Phys.* (Vol. 67, pp. 695–696).
5. Awasthi, A., Poonia, Y., Vaid, S., & **Sastri, O. S. K. S.** (2023). Alpha preformation probability from cluster formation model using AME-2020. In *Proceedings of the DAE Symp. on Nucl. Phys.* (Vol. 67, pp. 135–136).
6. Awasthi, A., & **Sastri, O. S. K. S.** (2023). Optimum number of parameters for np interaction using reference potential approach. In *Proceedings of the DAE Symp. on Nucl. Phys.* (Vol. 67, pp. 697–698).
7. Sharma, A., Sharma, A., & **Sastri, O. S. K. S.** (2023). Study of n-^{16}O using phase function method. In *Proceedings of the DAE Symp. on Nucl. Phys.* (Vol. 67, pp. 849–850).
8. Kant, I., Barbie, Sharma, J., & **Sastri, O. S. K. S.** (2023). Viola-Seaborg coefficients for partial alpha half-lives based on latest NNDC data. In *Proceedings of the DAE Symp. on Nucl. Phys.* (Vol. 67, pp. 271-272).
9. Awasthi, A., Sharma, A., & **Sastri, O. S. K. S.** (2024). Estimating Scattering Potentials for Nucleon-Nucleon Scattering using Physics Informed Machine Learning. In *Proceedings of the DAE Symp. on Nucl. Phys* (Vol. 68, p. 451).
10. Awasthi, S., Khachi, A., & **Sastri, O. S. K. S.** (2024). Study of elastic neutron-^{12}C scattering using Phase Function Method. In *Proceedings of the DAE Symp. on Nucl. Phys* (Vol. 68, p. 535).
11. Sharma, A., Awasthi A., Sharma, J., & **Sastri, O.S.K.S.** (2024). Study of 2+ Resonance in 4He (d, d) 4He using Reference Potential Approach. In *Proceedings of the DAE Symp. on Nucl. Phys* (Vol. 68, p. 667).
12. Kant, I., Awasthi, A., Sharma, A., & **Sastri, O. S. K. S.** (2024). Study of p-^{12}C at Astrophysical Energies using Phase Function method. In *Proceedings of the DAE Symp. on Nucl. Phys* (Vol. 68, p. 761).

Research in Indian Knowledge System in context of NEP-2020:

To promote research in the Indian Knowledge System, I have:

- Organized a Two-Day National Conference on "Indian Knowledge Systems in Physics and Astronomy" at IUAC, New Delhi.
- Attended the Viśva Veda Vijñāna Sammelanam, the 4th World Congress of Vedic Science, held at SASTRA Deemed University, Tamil Nadu.

The proceedings of these conferences are as follows:

1. Sastri, O. S. K. S., & Mishra, B. (2024). *Reflections on the Laws of Motion in Vaiśeṣika Darśanam*. In the Proceedings of the Two-Day National Conference on "Indian Knowledge Systems in Physics and Astronomy", IUAC, New Delhi.
2. Sastri, O. S. K. S. (2024). *GOD is Omnipresence, Omniscience, and Omnipotence: A Comprehension Based on the Concept of Fourier Transform*. In the Proceedings of the Viśva Veda Vijñāna Sammelanam, 4th World Congress of Vedic Science, SASTRA Deemed University, Tamil Nadu.

Research Projects:

To achieve the vision of establishing CUHP as a leading research university, two research projects have been initiated, laying the foundation for academic excellence and innovation.

1. An MoU with the **National Remote Sensing Agency (NRSA)** has been initiated, marking a significant step forward in advancing research capabilities. Under this collaboration, a Lightning Detection Sensor System has been installed at CUHP, inaugurated by the Honorable Vice-Chancellor. This project, supported by an annual funding of ₹1 lakh, is envisioned as a long-term initiative with a projected duration of 15–20 years, contributing to advancements in atmospheric and remote sensing research.
2. I have been assigned the role of **Chief Editor of the Physics Education Journal** has been sanctioned with a funding of ₹3.5 lakhs per annum for a duration of 5 years, amounting to a total allocation of ₹17.5 lakhs. This project aims to enhance the quality of physics education and contribute to the dissemination of innovative teaching and research practices in the field.

Constructing inverse scattering potentials for charged particles using a reference potential approach

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An accurate way to incorporate long-range Coulomb interaction alongside short-range nuclear interaction has been a challenge for theoretical physicists. In this paper, we propose a methodology based on the reference potential approach for constructing inverse potentials for charged particle scattering. The central idea is to obtain the inverse potential directly from the expected scattering phase shifts by comparing them with those obtained by solving the phase equation for a chosen reference potential. The design of the reference potential is key to incorporating the Coulomb interaction successfully. Here, a combination of two smoothly joined Morse functions, one regular followed by an inverted one, is considered. While the former takes care of short-range nuclear and Coulomb interactions, the latter accounts for expected barrier height due to the long-range Coulomb part that dominates once nuclear interaction subsides. The final step is to incorporate the phase equation within an iterative loop of an optimization algorithm to obtain the model parameters for the reference potential by minimizing the mean absolute percentage error between the obtained and expected scattering phase shifts. We have applied the methodology to the $\alpha - \alpha$ system and constructed the inverse potentials for its S, D, and G states with mean absolute percentage errors of 0.9, 0.5, and 0.4 respectively. Their respective resonances (experimental), in MeV, are found to be at 0.1240 (0.0918), 2.95 (3.03), and 11.89 (11.35). One can conclude that the reference potential approach using a combination of smoothly joined Morse functions is successful in accurately accounting simultaneously for the short-range nuclear and the long-range Coulomb interactions between charged particles in nuclear scattering studies.

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I. INTRODUCTION

The key to scattering phenomena is to model the underlying interaction potential that gives rise to the scattering phase shifts (SPS) that are responsible for the observed experimental scattering cross sections. The theoretical approaches [1–3] most often utilized rely upon determination of the scattering phase shift from the wave function that is obtained by solving the time independent Schrödinger equation (TISE). The potential is chosen by modeling the interactions primarily due to nuclear and Coulomb forces and, in some cases, by adding perturbation terms due to the interplay of spin, isospin, and orbital angular momentum. These potentials are typically represented by various mathematical functions that best represent the nature of the interaction as can be elicited from the phase shift values and the trends they follow at different laboratory energies [4]. An alternative approach is to rephrase the second order TISE as a first order nonlinear Ricatti equation for different ℓ channels as in the phase function method [5,6]. One advantage of this latter method is that it deals with the interaction potential directly, eliminating the requirement for a wave function. This enables the construction of inverse potentials directly from the available experimental data, as in inverse scattering theory [7].

Theoretically, constructing inverse potentials [8] requires not only information regarding all the bound state energies E_n $(n = 0, 1, \ldots, N)$ along with their related normalization constants C_n but also the phase shifts for all scattering energies $E > 0$ ranging to infinity. Most often, phase shift data are available for only certain energies within a limited range and hence a rigorous solution of the quantum mechanical inverse problem is extremely difficult to compute. The inverse problem is akin to the machine learning (ML) paradigm wherein one obtains the model of interaction from large amounts of available data. Typically, one prefers neural-network-based models [9], when the number of available experimental data is very large, say $\geqslant$1000. Otherwise, it is more appropriate to use metaheuristic algorithms [10–12] as part of these ML models and enhance their performance by incorporating the knowledge of the problem from the physics that underlies the phenomenon.

Selg [13] proposed using Morse functions as the zeroth reference to obtain the scattering phase shift, and went on to solve the Marchenko integral equation that gives rise to the inverse potential. We proposed a computational approach to construct inverse potentials [4,7] for nucleon-nucleon by utilizing a single Morse function as a reference. Although this method is effective in calculating the inverse potentials for scattering in scenarios where the projectile or target particles

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High-precision inverse potentials for neutron-proton scattering using piece-wise smooth Morse functions*

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Abstract: The aim of this study is to construct inverse potentials for various ℓ-channels of neutron-proton scattering using a piece-wise smooth Morse function as a reference. The phase equations for single-channel states and the coupled equations of multi-channel scattering are solved numerically using the 5th order Runge-kutta method. We employ a piece-wise smooth reference potential comprising three Morse functions as the initial input. Leveraging a machine learning-based genetic algorithm, we optimize the model parameters to minimize the mean-squared error between simulated and anticipated phase shifts. Our approach yields inverse potentials for both single and multi-channel scattering, achieving convergence to a mean-squared error $\leq 10^{-3}$. The resulting scattering lengths "a_0" and effective ranges "r" for $^3S_1$ and $^1S_0$ states, expressed as [a_0, r], are found to be [5.445(5.424), 1.770(1.760)] fm and [–23.741(–23.749), 2.63(2.81)] fm, respectively; these values are in excellent agreement with experimental ones. Furthermore, the calculated total scattering cross-sections are highly consistent with their experimental counterparts, having a percentage error of less than 1%. This computational approach can be easily extended to obtain interaction potentials for charged particle scattering.

Keywords: inverse potentials, neutron-proton scattering, piece-wise smooth Morse function, phase function method, reference potential approach, genetic algorithm

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I. INTRODUCTION

Current high-precision nucleon-nucleon potentials, available for scattering data up to the pion-threshold of 350 MeV, are provided by various groups; these include the Argonne v_{18} [1], Bonn [2], Reid [3], Nijmegen [4], and Paris [5] potentials. These potentials are modeled such that the NN interaction comprises one pion exchange potential for long inter-nuclear distances of $r \geq 2$ fm. The main differences between these high precision potentials stem from the way the nucleon-nucleon interaction is modeled for intermediate/medium (1.0 fm < r < 2 fm) and short-ranges (r < 1.0 fm) [6]. This modeling is performed using a central potential along with an interplay of orbital, tensor, spin-orbit, and quadratic spin-orbit terms. The approach involves simultaneously solving for the wave-functions based on the model potential and optimizing approximately 40–50 parameters for obtaining the phase shifts for all ℓ-channels for nucleon-nucleon scattering, from which the total cross sections are predicted to match the experimental ones [7]. An alternative methodology is to construct the inverse potentials utilizing the phase function method or variable phase approximation [8–10], which has the advantage of obtaining phase shifts by directly utilizing the potential, without the wave-function. Here, the second-order time-independent Schrödinger equation is transformed into a set of independent first-order non-linear Ricatti equations for each ℓ channel. Thus, one can determine the potentials corresponding to phase shifts for individual channels. This methodology is equivalent to constructing the model potential directly from the available scattering phase shifts data, which is the basic premise of the machine learning paradigm [11, 12].

Ideally, to obtain a complete inverse scattering solution, N discrete bound state energies $E_n < 0$ (n = 1, 2, …, N) and all possible scattering phase shifts for energies $E >$ 0, up to ∞, are required [13]. However, the available experimental data are limited to very few projectile energy values. Hence neural network-based machine learning models are not suitable, and we propose to utilize meta-

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Comparative study of α - α interaction potentials constructed using various phenomenological models

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Abstract: In this paper, we have made a comparative study of $\alpha-\alpha$ scattering using different phenomenological models like Morse, double Gaussian, double Hulthén, Malfliet-Tjon, and double exponential for the nuclear interaction, and atomic Hulthén as the screened Coulomb potential. The phase equations for S, D, and G channels have been numerically solved using the 5^{th}-order Runge-Kutta method to compute scattering phase shifts (SPS) for the elastic scattering region consisting of energies up to 25.5 MeV. The model parameters in each of the chosen potentials were varied in an iterative fashion to minimize the mean absolute percentage error (MAPE) between simulated and expected SPS. A comparative analysis revealed that all the phenomenological models result in exactly similar optimized potentials with closely matching MAPE values for S, D, and G states. One can conclude that any mathematical function that can capture the basic features of two-body interaction will always guide the construction of optimized potentials correctly.

Key words: α -α scattering, phenomenological models, screened atomic Hulthén, scattering phase shifts, resonance energies

1. Introduction

Scattering studies of α particles with ^4_2He nuclei are important for understanding the nature of nuclear force and gaining insights into few-body [1] and cluster models [2, 3]. Rutherford and Chadwick were the first to study α-α scattering in 1927 [4] and since then, numerous experiments have been performed at various energy levels to deepen our understanding. In 1956, Heydenburg and Temmer presented experimental scattering phase shifts (SPS) for the low-energy range of 0.6 MeV to 3 MeV [5]. Tombrello and Senhouse, in 1963, provided experimental SPS covering the energy range of 3.84 MeV to 11.88 MeV [6]. Nilson et al., in 1958, reported SPS for energies between 12.3 MeV and 22.9 MeV [7]. Subsequently, Chien and Brown, in 1974, contributed experimental SPS for the energy range of 18 MeV to 29.50 MeV [8].

The SPS data obtained from these experiments were compiled by Afzal et al. [9], which is generally considered by theoretical physicists for studying $\alpha-\alpha$ scattering. However, it is worth noting that their compilation included data only up until 1969. Recognizing the significance of incorporating Chien and Brown data from 1974, Khachi et al. updated the database for α-α scattering in 2022 [10].

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ORIGINAL PAPER

Elastic scattering study of α- ^{3}H and α- ^{3}He modeled by Malfliet–Tjon potential using phase function method

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Abstract: The interaction potentials for the resonant states of elastic scattering of $\alpha - ^3H$ and $\alpha - ^3He$ systems have been obtained using phase function method. While the interaction for nuclear part has been chosen as the Malfliet–Tjon (MT) form, the infinite-range Coulomb repulsion has been modeled using Hulthén, providing a screening effect to cut off the potential. The scattering phase shifts have been obtained by solving the phase equation for $\ell = 3$ using RK-5 method. The interaction parameters are optimized iteratively to minimize the mean absolute percentage error between the obtained and expected scattering phase shifts. Finally, the resonance energies for the $[5/2^-, 7/2^-]$ states of $\alpha - ^3H$ and $\alpha - ^3He$ systems have been obtained(experimental) in MeV, from the plots of their partial cross-sections, yielding [2.20 (2.18), 4.18 (4.14)] and [2.98 (2.98), 5.02 (5.14)] respectively. It can be concluded that the interaction potentials obtained using a combination of MT and Hulthén satisfactorily explain the observed scattering data.

Keywords: Scattering phase shifts; Malfliet–Tjon potential; Hulthén potential; Phase function method; RK-5 method; Elastic scattering; α-^{3}H scattering; α-^{3}He scattering; Partial Scattering cross section

1. Introduction

Abundance of most chemical elements in the Universe has been created by nucleosynthesis processes. Among these elements, heavier ones were produced by nuclear reactions, while lighter ones such as hydrogen, helium and lithium were generated during the Big Bang. The primordial abundance of ^7Li in the universe [1] is determined by $^3He(\alpha,\gamma)^7Be$ and $^3H(\alpha,\gamma)^7Li$ radiative-capture processes. These are also competing reactions in the proton-proton chain of solar hydrogen burning and are astronomically significant [2]. The production of ^7Be and ^{8}B neutrinos in the pp-II and pp-III branches is also determined by these reactions. Both these reactions are crucial for obtaining the correct fraction of proton-proton (*pp*) branches resulting in ^{7}Be versus ^{8}B neutrinos [3].

The important input information required for premordial nucleosynthesis is the nuclear reaction rate $N_A\langle\sigma v\rangle$, and $N_A\langle\sigma v\rangle$ which further depends on the velocity-averaged cross section (σ) of the nuclear reaction. The total cross section of the nuclear reaction is also required for obtaining the astrophysical S-factor. The astrophysical S-factor can be obtained experimentally, but for most of the thermonuclear reactions, it is feasible only at low energies, specifically between 100 KeV to 1 MeV [4, 5]. To properly describe any nuclear reaction existing in the Universe, the astrophysical S-factor requires considering electromagnetic transitions between different nuclear states.

From a nuclear physics viewpoint, these reactions are also important [6] because both odd-A nuclei, ^3He and ^3H are magic nuclei. After interaction with α particles, they form compound nuclei ^7Li and ^7Be both of which are again odd-A magic nuclei. The interaction between the valence nucleons in odd-A mirror nuclei can be determined by their scattering because the nuclei have similar inner structure and only differ by their valence nucleons. The entrance channels for both the reactions also consist of scattering of spin-$\frac{1}{2}$ particle with a spin-0 particle [7].

The scattering process is crucial for understanding the structure of the nucleus and the nature of interaction. Scattering phase shifts (SPS) indicate the strength of the interaction, and the scattering cross-section reveals the

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PARTIAL DIFFERENTIAL EQUATIONS

Algorithm to Obtain Inverse Potentials for α–α Scattering Using Variable Phase Approach

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Abstract—An algorithm has been developed with the purpose of obtaining inverse potentials, where the Riccati-type non-linear differential equation, also called phase equation, has been kept in tandem with the Variational Monte Carlo method. The optimization of Gaussian function parameters is achieved such that the experimental phase shifts are reproduced. The obtained SPS for various ℓ channels has been compared with experimental ones with mean absolute percentage error (MAPE) as a measure. The model parameters have been optimized by suitable optimization technique by looking for minimum value of MAPE. The results for $\ell = 0^+$, 2^+, and 4^+ partial waves have been obtained, to match with experimental SPS, with MAPE values of 2.9, 4.6, and 6.2, respectively, for data up to 23 MeV, while for higher states 6^+, 8^+, and 10^+ has MAPE of 3.2, 4.5, and 5.9, respectively, for data from 53–120 MeV. On extrapolation for data in range $E_{\ell ab} = 23$–120 MeV, using the optimised parameters, the SPS are found to be in close agreement with experimental ones for the first three channels.

Keywords: inverse potentials, alpha-alpha scattering, variable phase approach (VPA) or phase function method (PFM), scattering phase shifts, Double Gaussian potential

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1. INTRODUCTION

As stated by Kabanikhin in the 1950s, the first publication about inverse problems emerged in fields like physics (electrodynamics, quantum scattering theory, and acoustics), geophysics (electro-, seismo-, and geomagnetic exploration), and astronomy [1]. With the advent of powerful computers, these problems found use in almost every academic area that uses mathematical models, including medicine, industry, ecology, economics, linguistics, and social sciences. In recent years, there has been a growing interest in the study of heavy ion collisions and scattering problems, driven by their relevance in various fields of physics. Several computational approaches have been developed to tackle the complex nature of these problems, ranging from symbolic-numerical algorithms to numerical solutions. One notable contribution comes from Gusev et al. (2023) [2], who proposed a symbolic-numerical algorithm implemented in Maple for solving the inverse problem in the optical model for scattering in heavy ion collisions. Another important reference is the work by Puzynina and Vo Trong Thach [3], which offers a numerical solution for both direct and inverse scattering problems across various potentials. The core purpose of this paper is to investigate scattering of alpha particles which are in relative motion for all partial waves using an inverse approach. Rutherford and Chadwick were the first who experimentally studied α–α scattering in the year 1927 and since then a large amount of experimental data is available given by (i) Afzal et al. [4], (ii) S. Chien and Ronald E brown [5], (iii) Igo [6], (iv) Darriulat, Igo, Pug, Holm [7], (v) Nilson [8] and others. Alpha-alpha problem has been extensively studied both experimentally and theoretically with alpha particle having some sole properties like (i) zero spin and isospin (ii) tight binding energy of 28.3 MeV having property to form cluster-like states for lighter nuclei (^{6,7}Li, ^{9}Be, ^{12}C, and ^{16}O are α-structured) with alpha particle being the core nuclei in the cluster (iii) small root mean square radius of 1.44 fm.

In the 1940's for α–α scattering experiments, only naturally occurring α-sources like polonium, thorium and radium were used, which did not result in very accurate results from experiments. Later on, with the advancement of technology, accelerators were used in scattering processes and highly accurate phase

Simulation Study of Arbitrary 1D Periodic Potentials by Modified Marsiglio's Matrix Approach using Gnumeric Spreadsheet*

Aditi Sharma, Arushi Sharma and O S K S Sastri

In this paper, we present a structured approach to solving the quantum mechanical problem of a particle in 1D periodic square well potential to obtain energy eigenvalues. Here, we utilize the modified Marsiglio's matrix mechanics method to obtain energy eigenvalues for the potential. The method is implemented in a Gnumeric spreadsheet environment, a free open source software that has an in-built eigensolver, to gain clarity regarding each of the steps involved by choosing a smaller-sized H-matrix. The resultant eigenvalues obtained using the simulation are plotted with respect to k values to visualize the band structure diagrams arising from various periodic potentials such as a harmonic oscillator, inverted harmonic oscillator, and linear well potential. This implementation strategy helps us study the physical system in a more systematic and simpler way, pedagogically.

Aditi Sharma is an Assistant Professor at Chandigarh Group of Colleges, Jhanjeri, Mohali, Punjab. She completed her PhD in Physics from Central University of Himachal Pradesh, Dharamshala. This article presents part of work carried by her during PhD under Prof. O S K S Sastri.

Arushi Sharma is currently pursuing a PhD at the Central University of Himachal Pradesh under Prof. Sastri. Her study focuses on producing inverse potentials for nuclear charge configurations.

Sastri is a Professor at the Central University of Himachal Pradesh. His specialization includes computational physics, nuclear structure studies of actinides and superheavy nuclei, and specifically in physics education research.

1. Introduction

The study of periodic potentials is essential in many branches of physics, including solid-state physics, quantum mechanics, and condensed matter physics. Understanding how particles or waves behave in periodic potentials allows us to learn about material properties, electronic band structures, and transport phenomena. While analytical solutions exist for simple periodic potentials [1], studying more complicated and arbitrary potentials frequently necessitates numerical approaches and computational tools. The matrix method, notably Marsiglio's matrix approach [2], is a popular numerical approach that has been widely utilized to in-

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ELEMENTARY PARTICLES AND FIELDS
Theory

Real and Imaginary Phase Shifts for Nucleon–Deuteron Scattering Using Phase Function Method

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Abstract—The neutron–deuteron (nd) and proton–deuteron (pd) scattering are the simplest nucleon–nucleus scenario which throws light on understanding few body systems. In this work, real and imaginary parts of scattering phase shifts (SPS) for nd- and pd-scattering are obtained using complex potential, with Malfliet–Tjon (MT) model of interaction, by phase function method (PFM). The SPS for doublet $^2S_{1/2}$ and quartet $^4S_{3/2}$ states of nd- and pd-systems have been obtained for real and imaginary parts separately by solving the phase equation for $\ell = 0$, using Runge–Kutta 5^{th} order technique for laboratory energies up to 19 MeV. The obtained (real, imaginary) SPS for $^2S_{1/2}$ and $^4S_{3/2}$ states are matching with standard data with mean absolute error (MAE) of (1.32, 0.06) for $^2S_{1/2}$ state and (0.19, 0.06) for $^4S_{3/2}$ state of nd-scattering, (0.95, 0.28) for $^2S_{1/2}$ state and (0.58, 0.21) for $^4S_{3/2}$ state of pd-scattering.

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1. INTRODUCTION

One of the most crucial methods for determining the nature of interactions between various particles or systems of particles, which in turn would give information about their internal structure, is scattering experiments. The scattering phase shifts (SPS) obtained from phase wave analysis carry the specifics of the interaction for various ℓ-channels and are used to determine the differential and total scattering cross-sections (SCS). Quantum mechanically, the interaction potential between two particles is modeled and the corresponding time independent Schrodinger equation (TISE) is solved to obtain the wavefunction. The obtained wavefunction is matched with asymptotic free particle wavefunction, to determine SPS. A correct match between the obtained and experimental phase shifts validates the model of interaction potential.

Study of nucleon–deuteron scattering is one of the extensive fields of research and many research groups have studied it both theoretically and experimentally [1–5] over the years. For neutron–deuteron (nd) system, the SPS have been obtained using different phenomenological potentials such as Manning–Rosen [6] and Hulthén [7]. Alternatively, Faddeev equations have been solved for the three-body problem using AV18 nucleon–nucleon (NN) potential [8] and Malfliet–Tjon [9] potential [1, 5, 9]. The research on nd- and pd-systems has been focused both below [1–3, 5] and above the deuteron break up threshold of 3 MeV. G.L. Payne [1] et al. in 1980 solved Faddeev equations in configuration space to obtain ground state properties of Triton by considering NN interaction model of Malfliet–Tjon [9]. C.R. Chen et al. [2] solved configuration-space Faddeev equations for the nucleon–deuteron scattering below three-body breakup threshold to obtain scattering phase shifts and scattering parameters for doublet $^2S_{1/2}$ and quartet $^4S_{3/2}$ states of nd- and pd-systems.

Further, Ishikawa [5] studied interaction between proton and deuteron by a modified version of Faddeev equation to accomodate the Coulomb interaction and by considering Malfliet–Tjon [9] I–III NN-potential to obtain scattering phase shifts of nd- and pd-systems for lab energies ranging between 0.0015–3.27 MeV for nd-scattering and between 0.15–3.0 MeV for pd-scattering. A. Kievsky et al. [3] did critical phase shift analysis for nucleon–deuteron scattering for incident nucleon energy range from 1 to 3 MeV. For energies above deuteron breakup threshold, Kievsky et al. [10] applied complex Kohn variational principle and the hyperspherical harmonics technique and compared elastic nucleon–deuteron cross section with experimental data at several energies.

Z.M. Chen et al. [8] performed phase shift analysis for proton–deuteron for proton lab energies up to 22.7 MeV using AV18 NN-potential. Pisa group

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Cross Section for $^3H(\alpha,\gamma)^7Li$ Astrophysical Reaction Using Scattering Phase Shifts

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Abstract. The chemical abundance of different elements in Universe depends substantially on the nuclear structure and nuclear reactions. In order to determine the primordial ^7Li abundance in the early Universe, the $^3H(\alpha,\gamma)^7Li$ radiative-capture process is of great astronomical relevance. The calculations of primordial nucleosynthesis offer some thorough and comprehensive assessments of main assumptions of the big-bang model. The key information required for these calculations is nuclear reaction rate $N_A < \sigma v >$, which further depends on the velocity-averaged cross section (σ) of the nuclear reaction. Astrophysical S-factor calculations also require the total cross section of any reaction. The cross section for $^3He(\alpha,\gamma)^7Be$ Astrophysical reaction has already been obtained by our group for laboratory energies up-to 9 MeV. In this work, we have calculated cross section for the reaction $^3H(\alpha,\gamma)^7Li$ by calculating scattering phase shifts using Phase function method. The phase shifts are calculated for laboratory energies below 15 MeV for $\frac{5}{2}^-$ and $\frac{7}{2}^-$ resonant states of ^7Li (partial wave $\ell = 3$). The calculated resonance energies (E_r) and width of resonance Γ are: $E_r = 2.19\ MeV$ ($exp = 2.18\ MeV$), $\Gamma = 0.090\ MeV$ ($exp = 0.069\ MeV$) for $\frac{7}{2}^-$ resonant state and $E_r = 3.60\ MeV$ ($exp = 4.14\ MeV$), $\Gamma = 0.704\ MeV$ ($exp = 0.918\ MeV$) for $\frac{5}{2}^-$ resonant state.

1 Introduction

Previously [1] we have obtained scattering phase shifts (SPS), E_r and Γ using Morse +Coulomb+Spin-Orbit interaction for $\alpha - ^3 He$ system. Since, no significant effect of Spin-Orbit interaction was found in our previous work so only Morse+Coulomb interaction has been used as an interaction potential to obtained SPS for $\alpha - ^3 H$ system. The total interaction potential is utilised in phase equation [2]; which is solved numerically using RK-5 method; to obtain SPS with initial condition $\delta_\ell(k, 0) = 0$.

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Numerical Simulation Study of Light Nuclei

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Introduction

Understanding structure of light nuclei requires a systematic study of nucleon-nucleon, nucleon-nucleus and nucleus-nucleus scattering. Theoretically, interaction is modeled and Schrödinger equation is solved to undertake phase-wave analysis to obtain scattering phase shifts(SPS), which are utilised to explain observed cross-section and resonances.
Mostly, theoretical methods such as R-matrix, S-matrix, Jost and complex scaling methods focus on obtaining the wavefunction to determine the SPS. Alternatively, the phase function method utilise only the interaction potential to determine SPS for various ℓ-channels, by solving a first order non-linear differential equation using RK-methods.
While we have utilised Morse function as model of interaction, in our preliminary work, to study *nd*, *pd* systems [1–3] and $\alpha - ^3 H$, $\alpha - ^3 He$ systems, later we focused our research on testing the performance of Malfliet-Tjon(MT) potential. Even though this potential has been successfully utilised for study of *np* scattering and off-shell calculation of Triton binding energy, it was not explored for study of other light nuclei scattering. Another important objective of this research is to utilise optical potential model with MT potential to understand the nature of interaction for energies beyond elastic scattering threshold.

Research methods

Model of interaction:
The nuclear interaction: In elastic region, MT potential, which has a repulsion term of Yukawa form, given by:

$$V_{MT}(r) = -V_A\Big(\frac{e^{-\mu_A r}}{r}\Big) + V_R\Big(\frac{e^{-\mu_R r}}{r}\Big) \quad (1)$$

with, $\mu_R = 2\mu_A\ fm^{-1}$ is used.
The Coulomb interaction: For charged interacting systems, two screened Coulomb ansatzs have been employed:

$$V_C(r) = z_1 z_2 \frac{e^2}{r} erf(\beta r) \quad (2)$$

Here, $\beta = \frac{\sqrt{3}}{2\times R}\ fm^{-1}$, where R is root mean square radius and

$$V_C(r) = V_H(r) = V_0 \frac{e^{-r/a}}{1 - e^{-r/a}} \quad (3)$$

Here V_0 and a satisfy $aV_0 = 2k\eta$, where η is Sommerfeld parameter. Here, $k = \sqrt{\frac{E}{\hbar^2/2\mu}}$, with μ being reduced mass of system.
Optical potential model: For inelastic collision energies, imaginary part of SPSs are given, we choose potential to also have an imaginary part as follows:

$$V(r) = V^R_{MT}(r) + iV^I_{MT}(r) \quad (4)$$

Phase Function Method: The phase equation, obtained from radial time independent Schrödinger equation, is given by:

$$\delta'_\ell(k,r) = -\frac{V(r)}{k(\frac{\hbar^2}{2\mu})}\Big[\cos(\delta_\ell(k,r))\hat{j}_\ell(kr) - \sin(\delta_\ell(k,r))\hat{\eta}_\ell(kr)\Big]^2 \quad (5)$$

Here, δ_ℓ is phase shift of ℓ^{th} partial wave, $\hat{j}_\ell(kr)$ and $\hat{\eta}_\ell(kr)$ are the Riccati-Bessel and Riccati-Neumann functions for ℓ^{th} partial

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Modelling Long Range Coulomb interaction in pp-Scattering studies using Reference Potential Approach

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I. INTRODUCTION

Incorporating coulomb interaction into scattering calculations, involving charged prticles is a major challenge in nuclear and particle physics, due to its long range behaviour. Typically, screened coulomb potentials based on erf()[1] and atomic Hulthen[2] are utilised in PFM to explain observed scattering phase shifts with reasonable success. In this work we introduce a reference potential as a combination of smoothly linked Morse potentials as suggested by Selg[3], to model both nuclear and Coulomb interaction, by choosing to study S-state pp-scattering. The key idea is to incorporate an inverted Morse function at the long range to model the Coulomb potential.

II. METHODOLOGY

A. Reference Potential Approach

In this paper, for exactly solvable reference potential, three smoothly joined components of Morse potential for 3 different regions are choosen.

In first region, there is Pseudo Morse potential (say U_0), with $D_0 = \hbar^2{\alpha_0}^2/8m$. This signifies a limit depth for the potential, when there is no discrete energy spectrum any longer. The second region is a regular Morse potential(say U_1) which models both nuclear and Coulomb interactions. Finally, in 3rd region we fit a reversed morse component (say U_2) which accomodates long range Coulomb interaction with D_2 being negative. The overall potential is as follows:

$$U(r) = \begin{cases} V_0 + D_0[e^{-2\alpha_0(r-r_0)} - 2e^{-\alpha_0(r-r_0)}], & \text{if } r \leq X_1 \\ V_1 + D_1[e^{-2\alpha_1(r-r_1)} - 2e^{-\alpha_1(r-r_1)}], & \text{if } X_1 < r < X_2 \\ V_2 + D_2[e^{-2\alpha_2(r-r_2)} - 2e^{-\alpha_2(r-r_2)}], & \text{if } r \geq X_2 \end{cases} \quad (1)$$

For obtaing a smooth potential, the following boundary conditions at should be satisfied,

$$U_k|_{X_k} = U_{k+1}|_{X_k} \quad (2)$$

$$\frac{dU_k}{dr}|_{X_k} = \frac{dU_{k+1}}{dr}|_{X_k} \quad (3)$$

where k takes values 1 & 2. while U_0 has only 3 parameters due to D_0 dependence on α_0, both U_1 and U_2 have 4 parameters each. Further, X_1 and X_2 also need to be varied. So, it is a fairly complex potential with 13 parameters. Using the 4 boundary conditions, one obtains D_1, D_2, V_0, V_1 parameters of the potential. Hence, the total no of the model parameters reduces to 7. Now we choose to apply this approach to construct inverse potential for singlet state of pp-scattering using PFM.

B. Phase Function Method(PFM)

The final reference potential is utilised as input in the Phase equation for l=0 ($^1S_0 - state$), given by

$$\delta_0'(k,r) = -\frac{u(r)}{k}\sin^2(\delta_0 + kr) \quad (4)$$

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Phase Wave Analysis of Nucleon-α Scattering using Malfliet-Tjon Potential

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Introduction

The scattering study plays an important role in analyzing the structure of various nuclei and the corresponding interactions involved. The scattering of nucleons with stable α particles is an important study since the α particle is a stable nucleus and it has very high binding energy; greater than $\approx$ 20 MeV; and hence at low energies, the probability of inelastic scattering with α particle is negligible. Many experimental and theoretical groups [1, 2] have studied nucleon-α systems with reasonable success. In this work, we have obtained scattering phase shifts and interaction potentials for $p_{1/2}$ and $p_{3/2}$ states of $n\alpha$ and $p\alpha$ scattering.

The scattering phase shifts and inverse potentials [3] for elastic scattering of nucleons with α-particle are obtained by numerically solving phase equation in conjunction with optimisation technique. The phase equation which depends predominantly on the interaction potential, is solved using 5^{th} order Runge-Kutta (RK-5) method for the laboratory energies below 20 MeV. We have already obtained scattering phase shifts for nucleon-nucleon and nucleon-nucleus systems successfully [2–4]. Here, we are considering Malfliet-Tjon [5] potential as our interaction model for calculating nucleon-α system.

Methodology

The total interaction potential consist of Malfliet-Tjon (MT) potential and the Coulomb potential. The MT potential is a three parameter potential consisting of both attractive and repulsive parts given as:

$$V(r) = V_{MT}(r) = -V_A\Big(\frac{e^{-\mu_A r}}{r}\Big) + V_R\Big(\frac{e^{-\mu_R r}}{r}\Big) \tag{1}$$

with, $\mu_R = 2\mu_A\ fm^{-1}$. For proton-α ($p\alpha$) interaction, Coulomb term is added [3], given by:

$$V_C(r) = z_1 z_2 \frac{e^2}{r} erf(\beta r) \tag{2}$$

here $e^2 = 1.44\ MeVfm$, $z_1 z_2 = 2$ for $p\alpha$ system and $\beta = \frac{\sqrt{3}}{2\times R_{p\alpha}}$. Where, $R_{p\alpha} = 1.422fm$ [6] is root mean square (rms) radius of proton-α system, giving $\beta = 0.609\ fm^{-1}$. The total interaction potential for $p\alpha$ system given as:

$$V(r) = -V_A\Big(\frac{e^{-\mu_A r}}{r}\Big) + V_R\Big(\frac{e^{-\mu_R r}}{r}\Big) + (\frac{e^2}{r}) erf(0.609r) \tag{3}$$

To obtain the scattering phase shifts, the total interaction potential is incorporated in the phase equation [3], as:

$$\delta'_\ell(k,r) = -\frac{V(r)}{k(\frac{\hbar^2}{2\mu})}\Big[\cos(\delta_\ell(k,r))\hat{j}_\ell(kr) - \sin(\delta_\ell(k,r))\hat{\eta}_\ell(kr)\Big]^2 \tag{4}$$

Here, $k_{cm} = \sqrt{\frac{E_{cm}}{\hbar^2/2\mu}}$; with $\mu_{n\alpha} = (m_n * m_\alpha)/(m_n + m_\alpha) = 750.47$ and $\mu_{p\alpha} = (m_p * m_\alpha)/(m_p + m_\alpha) = 749.64$ in the units MeV/c^2 respectively are the reduced mass of $n\alpha$ and $p\alpha$ system.

$\hat{j}_\ell(kr)$ and $\hat{\eta}_\ell(kr)$ in Eq. (4) are the Riccati-Bessel and Riccati-Neumann functions for ℓ^{th} partial wave with condition $\delta_\ell(k,0) = 0$. Phase equation Eq. (4) for $\ell = 1$ partial wave

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Alpha preformation probability from Cluster Formation Model using AME-2020

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Introduction

Alpha decay, one of the most common decay modes of heavy and superheavy nuclei, has received a lot of attention since it can be used as a probe to examine unstable nuclei and it is the sole means to identify newly synthesized superheavy nuclei [1]. Recently, Cluster Formation Model (CFM), a quantum mechanical technique for determining preformation factor$P\alpha$ using binding energy differences of parent and daughter nuclei was presented, and it was successful in reproducing the microscopic calculation results for ^{212}Po [2]. Ahmed et al. successfully determine the α cluster preformation probability in even-even nuclei using this technique. Despite the fact that this has been expanded for odd-odd and odd A nuclei , as well as the study of α-decay, the analysis is limited to shell closures and does not reveal the relative stability of odd-N nuclei in comparison to their neighboring even-N nuclei. P.C. Sood et.al. [3] proposed a criterion for determining the relative longevity of odd-N nuclei in the actinide region as follows: *If three or more consecutive members of an isotopic sequence lying at or near the β-stability curve have α-emission as their dominant decay mode and if an intense favoured decay is available for the odd-N parent, then the odd-N member of the isotopic triad has generally a longer half-life than that of its even-N neighbour on either side, irrespective of Z being even or odd.* In this work, we will examine the alpha-cluster preformation factors for the actinide region to confirm the relative longevity of odd N nuclei compared to their even-N neighbors using experimental BEs from the atomic mass evaluation in AME2020 [4].

Theoretical Framework

In CFM, a nucleus with energy E consists of a daughter nucleus and an alpha particle with a formation energy of $E_{f\alpha}$, the α preformation factor can be expressed as $P_\alpha = \frac{E_{f\alpha}}{E}$. The Formation Energy of Cluster $E_{f\alpha}$ for different nuclei type are given as follows [5]:

Case 1: For Even Z - Even N nuclei

$$E_{f\alpha} = 3B(A,Z) + B(A-4,Z-2) \\ -2B(A-1,Z-1) - 2B(A-1,Z)$$

Case 2: : For Even Z - Odd N nuclei

$$E_{f\alpha} = 3B(A-1,Z) + B(A-5,Z-2) \\ -2B(A-2,Z-1) - 2B(A-2,Z)$$

Case 3: For Odd Z - Even N nuclei

$$E_{f\alpha} = 3B(A-1,Z-1) + B(A-5,Z-3) \\ -2B(A-2,Z-2) - 2B(A-2,Z-1)$$

Case 4: For Odd Z - Odd N nuclei

$$E_{f\alpha} = 3B(A-2,Z-1) + B(A-6,Z-3) \\ -2B(A-3,Z-2) - 2B(A-3,Z-1)$$

By using these formulae, one can calculate P_α using experimental Binding energies, which are taken from the latest atomic mass evaluation in AME2020.

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Alpha preformation probability from Cluster Formation Model using AME-2020

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Introduction

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Case 2: : For Even Z - Odd N nuclei

$$E_{f\alpha} = 3B(A-1,Z) + B(A-5,Z-2) \\ -2B(A-2,Z-1) - 2B(A-2,Z)$$

Case 3: For Odd Z - Even N nuclei

$$E_{f\alpha} = 3B(A-1,Z-1) + B(A-5,Z-3) \\ -2B(A-2,Z-2) - 2B(A-2,Z-1)$$

Case 4: For Odd Z - Odd N nuclei

$$E_{f\alpha} = 3B(A-2,Z-1) + B(A-6,Z-3) \\ -2B(A-3,Z-2) - 2B(A-3,Z-1)$$

By using these formulae, one can calculate P_α using experimental Binding energies, which are taken from the latest atomic mass evaluation in AME2020.

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Study of n-^{16}O using Phase Function Method

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Introduction

Investigating the reaction of radiative n-^{16}O capture is undoubtedly intriguing since it is linked to the sequence of thermonuclear reactions that took place during ancient nucleosynthesis and eventually give rise to the sun, stars and our cosmos[1]. Neutron radiation therapy calculations of energy deposition in the human body are particularly sensitive to the partial cross sections for neutron interactions with oxygen. Despite their practical importance , there have been very few measurements of oxygen scattering cross sections. In this paper, we have find the resonance energy for n-^{16}O using obtained phase shifts. Neutron induced scattering experiments are difficult to perform , but theoretically its much easier to study neutron scattering as it does not involve electromagnetic interaction. Therefore, there is no need for additional coulomb potential; a single phenomenological potential would enough to obtain the scattering phase shifts. Scattering Phase shifts have been calculated by Phase Function Method(PFM) by considering Morse potential as an interaction potential.

Methodology

As a zeroth approximation consider morse potential as:

$$V_N(r) = D_0 \left(e^{\frac{-2(r-r_m)}{a_m}} - 2e^{\frac{-(r-r_m)}{a_m}} \right) \quad (1)$$

where D_0 , r_m and a_m represent the depth of potential , equilibrium distance and shape of potential respectively. It is a three parameter potential. The time independent Schrödinger equation (TISE) can be written as

$$\frac{d^2u_\ell(r)}{dr^2} + \left[k^2 - \frac{\ell(\ell+1)}{r^2} - U(r) \right] u_\ell(r) = 0 \quad (2)$$

where $U(r) = V(r)/(\hbar^2/2\mu)$ & $k_{c.m} = \sqrt{E_{c.m}/(\hbar^2/2\mu)}$ The TISE in Eq.2 can been transformed to a non-linear Riccati equation of first order[2], which directly relates to scattering phase shifts(SPS) as given by

$$\delta'_\ell(k,r) = -\frac{U(r)}{k} \Big[\cos(\delta_\ell(k,r))\hat{j}_\ell(kr) - \sin(\delta_\ell(k,r))\hat{\eta}_\ell(kr) \Big]^2 \quad (3)$$

The Riccati Hankel function of first kind is given by $\hat{h}_\ell(r) = -\hat{\eta}_\ell(r) + i\,\hat{j}_\ell(r)$, where $\hat{j}_\ell(kr)$ is Ricatti-Bessel and $\hat{\eta}_\ell(kr)$ Riccati-Neumann function. By substituting the expressions for different ℓ-values of these two later functions, we obtain the respective phase equations as

$$\delta'_2(k,r) = -\frac{U(r)}{k} \Big[-\sin(\delta_2+\kappa) - \frac{3\cos(\delta_2+\kappa)}{\kappa} + \frac{3\sin(\delta_2+\kappa)}{\kappa^2} \Big]^2 \quad (4)$$

This equation is solved using 5th order Runge-Kutta methods by choosing the initial condition as $\delta_\ell(0,k) = 0$ and integrating it to a large distance. The significant advantage of PFM method is that the phase-shifts are directly expressed in terms of the potential and have no relation to the wavefunction. By using a suitable optimization technique[3] we have optimize the potential parameters to obtain the SPS's which are in good agreement with experimental SPS.

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Viola-Seaborg Coefficients for Partial Alpha Half-lives Based on Latest NNDC Data

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Introduction

The Geiger-Nuttall law is an empirical relationship that describes the behavior of alpha particle emission rates for heavy atomic nuclei. It reveals a straight line with a slope of A_Z and an intercept of B_Z when plotting the logarithmic graph of α half-life of given isotopes vs. the inverse square root of effective α energy. Viola-Seaborg (VS) coefficients, which are obtained from the graph between different atomic number nuclei and A_Z & B_Z, can be generalized into a single formula that can be used to calculate the α half-life of heavy nuclei.

In this work, we have recalculated the Viola-Seaborg coefficients[1] by observing total of 104 α-decay favoured nuclei for atomic number range of Z = 88 to 98, using the latest NNDC data[2], incorporating the effects of alpha transition intensity and electron screening correlation[3].

Methodology

In 1911, Geiger-Nuttel (GN) established a relationship between α-decay and partial half-life as follows:

$$\log_{10} T^{\alpha}_{\frac{1}{2}} = \frac{A_z}{\sqrt{E^*_\alpha}} + B_z$$

Here, A_z and B_z are Z-dependent coefficients, and $T^{\alpha}_{\frac{1}{2}}$ is the experimental half-life of α-decay and E^*_α is effective α-decay energy and is given by:

$$E^*_\alpha = \frac{A}{A-4}E_\alpha + \Delta E_{SC}$$

Here, ΔE_{SC} is the electron screening correction[4] and is expressed as:

$$\Delta E_{SC} = (6.5 \times 10^{-5}) Z_D^{\frac{7}{5}}\,\text{MeV}$$

Here, Z_D is atomic number of daughter nuclei. The partial half-life $T^{\alpha}_{\frac{1}{2}}$ of isotopic nuclei is given as:

$$T^{\alpha}_{\frac{1}{2}} = \frac{T_{\frac{1}{2}}}{\text{B.R.} \times I_\alpha}$$

Here, $T_{\frac{1}{2}}$ is the total half-life, B.R. is the branching ratio for α-decay and I_α is intensity of ground state to ground state transition. Viola-Seaborg(VS) generalized the GN law to the Viola-Seaborg Formula (VSF) in 1966 by observing straight and nearly parallel lines:

$$\log_{10} T^{\alpha}_{\frac{1}{2}} = \frac{aZ+b}{\sqrt{E^*_\alpha}} + (cZ+d) + h_{log}$$

Here a, b, c, and d are VS coefficients obtained from even-even nuclei, and $h_{\log}$ is the hindrance factor calculated for unpaired nucleons.

Result and Discussion

In this work, A_z and B_z were obtained by observing a regression line graph between $(E^*_\alpha)^{-\frac{1}{2}}$ and $\log T^{\alpha}_{\frac{1}{2}}$ for forty even-even nuclei (including their α-decay favoured isotopes) as

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Estimating Scattering Potentials for Nucleon-Nucleon Scattering using Physics Informed Machine Learning

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Introduction

The nucleon-nucleon (NN) interaction remains an open problem in nuclear physics. Groups like Argonne, Bonn, and Nijmegen have developed high-precision NN potentials using one-pion exchange at long range (r $\geq$ 2.0 fm) and various interaction terms like central, tensor, spin-spin, spin-orbit, tensor etc. at intermediate (1.0 < r < 2.0) fm and shorter ranges (r < 1.0 fm) to fit phase shifts [1]. Alternatively, the phase function method is way estimates potentials directly from scattering observables such as phase shifts. This is conceptually similar to machine learning. In this work, we apply a Physics-Informed Machine Learning (PIML) approach to estimate the inverse potential for the $^1S_0$ NN scattering state by minimizing the mean square error (MSE) between simulated and observed phase shifts using a piecewise Morse function as a reference model [2].

Methodology

We employ a PIML framework, integrating the phase function method to estimate the $^1S_0$ NN scattering potential [2].

A. Phase Function Method

The Schrödinger equation for $\ell = 0$ is transformed into a Ricatti-type equation:

$$\delta_0'(k,r) = -\frac{V(r)}{(\hbar^2/2\mu)k}\sin^2[kr+\delta_0(r)], \qquad (1)$$

where $k_{c.m} = \sqrt{E_{c.m}/(\hbar^2/2\mu)}$. This non-linear equation is solved using the RK-5 method with initial condition $\delta_0(k,0) = 0$. Using phase shifts as input, the potential is estimated with a piecewise Morse function as the reference model [2].

B. Optimization Using Machine Learning

We optimize the model parameters of the reference model [2] using machine learning based algorithm such as genetic algorithm (GA) [3]. GA is versatile, suitable for optimizing discontinuous, non-differentiable, stochastic, or highly non-linear functions. Moreover, GAs are easily parallelizable, fast, and capable of exploring vast search spaces efficiently. They can accommodate multiple complex optimization objectives. Using this algorithm, we optimized the model parameters by minimizing the loss function called MSE, defined as

$$MSE = \frac{1}{N}\sum_{i=1}^{N}(\delta^i_{inp}(kr) - \delta^i_{obt}(kr))^2,$$

Here, $\delta^i_{inp}(kr)$ represents the input phase shifts from the Wiringa *et. al.* [4], while $\delta^i_{obt}(kr)$ are the optimized values obtained by solving the phase equations. The input phase shifts, defined at various energies and angular momenta ℓ, are used to infer the unknown potential $V(r)$. By optimizing the model parameters with GA, we reconstructed the inverse potentials. The optimization is guided by the phase function method to ensure physical consistency. The Physics-Informed Machine Learning approach integrates physics into the process, effectively identifying the best-fit parameters for the inverse scattering potential.

Results and Discussion

The scattering phase shift data up to 350 MeV for the $^1S_0$ state of neutron-proton, proton-proton, and neutron-neutron interactions, provided by Wiringa *et al.* [4], is used as input to

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Study of $p-^{12}$C at Astrophysical Energies Using Phase Function Method

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Introduction

The p-^{12}C reaction is crucial for energy production in stars, particularly through the CNO cycle [1]. Studying this reaction in the astrophysical energy range (0–10 keV) provides key insights into stellar processes. In this work, we model the interaction potential for proton energies between 400 keV and 1000 keV in the elastic region, focusing on scattering from the $^2S_{1/2}$ ($\ell = 0$) resonant state, which predominantly contributes to the astrophysical S-factor. The inverse potentials are constructed using the phase function method (PFM), which directly obtains phase shifts from the potential without relying on the wave function. The nuclear interaction between the proton and ^{12}C is modeled using the Morse potential, while the screened Coulomb interaction is described by the atomic Hulthén potential.

Methodology

A. Modeling the Interactions

The nuclear part of the $p-^{12}$C interaction, is modeled by Morse potential and the long-range Coulomb interaction, which gets typically screened in experimental scenario, is taken to satisfy the Atomic Hulthén form of the potential [2]. hence, the net model of interaction is given by

$$V(r) = V_0\left(e^{\frac{-2(r-r_m)}{a_m}} - 2e^{\frac{-(r-r_m)}{a_m}}\right) + V\frac{e^{-r/a}}{1-e^{-r/a}}$$

where V_0 represents the nuclear potential strength, a_m is the shape parameter, r_m is the equilibrium distance, V is the strength of the Coulomb barrier and a is the screening parameter. V and a are related to the Sommerfeld parameter η as:

$$aV = 2k\eta = \frac{2Z_1Z_2e^2\mu c^2}{\hbar^2c^2} = 0.384 \text{ fm}^{-1}$$

B. Phase Function Method (PFM)

The second order time-independent Schrödinger equation for $\ell = 0$ is transformed into nonlinear Riccati equation, given as:

$$\delta_0'(k,r) = -\frac{U(r)}{k}\sin^2[kr + \delta_0(r)]$$

where $k = \sqrt{E_{c.m}/(\hbar^2/2\mu)}$. The Runge-Kutta method is employed to solve this equation using the initial condition $\delta_0(k,0) = 0$.
Mean Absolute Percentage Error (MAPE) is used as the cost function in the optimization procedure driven by Genetic algorithm [3], which is given as:

$$\text{MAPE} = \frac{1}{N}\sum_{i=1}^{N}\left|\frac{\delta^i_{inp} - \delta^i_{sim}}{\delta^i_{inp}}\right| \times 100$$

where δ^i_{inp} represents the expected phase shift values[1], and δ^i_{sim} represents the calculated values from our simulation procedure.

Result and Discussion

The available scattering phase shift data for the S-state of $p-^{12}$ C ranges a little below 400 keV to 1 MeV. There are only six data points that capture the sharp variation in phase shifts from 400 to 500 keV and the rest of the 25 data points which are closely spaced between 500 to 1000 keV fall almost on a horizontal line with an extremely small negative slope. Trying to consider all the points in the data during optimization leads to poor convergence in the rising part of the phase

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Study of 2^+ Resonance in $^4He(d,d)^4He$ using Reference Potential Approach

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Introduction

Scattering studies between light nuclei is the only way by which one can understand the level structure of the nucleus with small Z values. Analyzing $^4He(d,d)^4He$ elastic scattering data by considering energy-dependent phase shifts gives rise to the energy levels of ^6Li nuclei. Deuteron-alpha (dα) scattering is also of interest in connection with the problem of measuring the spin polarization parameters characterizing deuteron beams. Over the past few decades, $^4He(d,d)^4He$ scattering has been extensively investigated for the low-lying levels of ^6Li, both experimentally and theoretically [1, 2]. In this study, we determine the underlying interaction potential for the resonance states of $^3D_1$ and $^3D_2$ of the dα system by utlizing the reference potential approach[3]. The obtained scattering phase shifts from the underlying potentials are utilized to calculate the respective partial cross-sections and resonance energies.

Methodology

The phase equation for $^3D_1$ and $^3D_2$, with $\ell = 2$, is given by

$$\delta_2'(k,r) = -\frac{U(r)}{k}[-\sin(\delta_2 + kr) - \frac{3\cos(\delta_2+kr)}{kr} + \frac{3\sin(\delta_2+kr)}{kr^2}]^2 \quad (1)$$

where one needs to provide a mathematical function to represent the form for the potential U(r). This will be constructed as a combination of three Morse functions over three different regions of interaction, with the third Morse function being negative to account for the Coulomb interaction [3], connected in a piece-wise smooth manner. Each Morse function comprises of 4 parameters which results in 12 parameters in addition to two boundary points. By applying the continuity conditions for the three Morse functions and their derivatives at the two boundaries, one can solve the resulting four equations and obtains 4 of the parameters to be dependent on other parameters. Hence, the total number of the model parameters is reduced to 10. Varying the two boundary points further provides flexibility and hence greater variety. The 10 parameter reference potential is given as input to solve the phase equation, (eqn. ref1), numerically using RK-5 method with the initial condition as $\delta_2(k,0) = 0$. The final integration distance can be chosen as far as possible. The phase equation is solved by updating the model parameters in each iteration, using a genetic algorithm[3], so as to minimize mean absolute percentage error (MAPE) between the obtained and expected scattering phase shifts.

Results and Discussion

Varying the parameters of the reference function will produce a wide range of curves in the sample space. While, the inverse potential that best matches the expected phase shifts is determined by optimizing the reference function's parameters to minimize a cost function.

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Study of elastic neutron-^{12}C scattering using Phase Function Method

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Introduction

It is a well established fact that for light nuclei at low energies, the sharply defined resonance peaks reveal important information about the interaction between a nucleon and a nucleus [1]. The study of interaction between a neutron and ^{12}C isotope is very important since ^{12}C is an abundant isotope of carbon in nature ($\approx$ 98.9 %) and studying the interaction helps researchers to understand the nature of nucleon-nucleus interaction, the internal level structure of the ^{13}C isotope, binding energy and the nuclear reactions occuring in stars and the underlying nuclear forces occuring. It is also the final product of triple-alpha process occuring in the cores of stars, in which the three ^{4}He nuclei are fused to form a ^{12}C nucleus by releasing considerable amount of energy. One another interesting fact about ^{12}C is that it is a simple isotope of carbon with equal number of protons and neutrons, and is also quite stable with relatively very high binding energy per nucleon equal to 7.68 MeV. The ^{12}C plays a cruicial role in carbon-nitroge-oxygen (CNO) cycle also which is essential for understanding the life cycle of a star and and the energy production in red giant stars [2].
Interaction of neutron with ^{12}C nucleus (n-^{12}C) has been studied by many researchers experimentally and theoretically [1–3]. In this work, we are studying the interaction by determining scattering phase shifts and interaction potentials for s, p, d and f states. Experimental measurements for n-^{12}C in energy ranges from 1.45 to 7 MeV reveals resonances at neutron bombarding energies of 3.5, 4.23, 4.93 and 6.29 MeV. A Phase Function Method approach have been carried out to obtain the interaction potentials for n$-^{12}C$ elastic scattering for different channels in energy range 1.45-7 MeV [2, 3].

Methodology

The interaction between neutron and ^{12}C is modeled by considering Malfliet-Tjon (MT) [4] potential as the interaction potential. The MT potential is a three parameter potential consisting of both attractive and repulsive parts given as:

$$V(r) = -V_A\Big(\frac{e^{-\mu_A r}}{r}\Big) + V_R\Big(\frac{e^{-\mu_R r}}{r}\Big) \quad (1)$$

with, $\mu_R = 2 \times \mu_A$. The interaction potential is incorporated in the phase equation [4], as:

$$\frac{d\delta(k,r)}{dr} = -\frac{V(r)}{k}\frac{2\mu}{\hbar^2}\Big[\cos(\delta_\ell(k,r))\hat{j}_\ell(kr) - \sin(\delta_\ell(k,r))\hat{\eta}_\ell(kr)\Big]^2 \quad (2)$$

Here left side of equation presents differentiation of phase function with respect to distance. Also wavenumber $k_{cm} = \sqrt{\frac{E_{cm}}{\hbar^2/2\mu}}$; with $\mu_{n^{12}C}$ is the reduced mass of n-^{12}C system given by $(m_n * m_{^{12}C})/(m_n + m_{^{12}C}) = 866.81$ MeV/c^2 and $\hbar^2/2\mu$ for n-^{12}C system is equal to 22.46 MeV-fm^2. For higher partial waves (p, d & f) the Riccati-Bessel ($\hat{j}_\ell$) and Riccati-Neumann

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Reflections on Laws of Motion in Vaiśeṣika Darśanam

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Abstract

***Introduction**:*

The evolution of Science [1] has been possible due to the invention of tools with increasing capabilities and their use to observe phenomena. There are two types of tools:

1. *Instruments that allow us to observe the underlying patterns in the physical universe, and*
2. *Symbolic systems for representation of the observed patterns that lead to explanations*

The cognitive capabilities of human beings have constantly evolved in the form of creating, remembering, and modifying mental models to be able to cope with the environment. So, model building is central to common sense thinking of humans. This capacity has been further expanded and augmented by the creation of semiotic systems. These are representational systems in the form of signs such as symbols, tokens, icons, diagrams, etc. The most notable advance is the spoken and written language.

In the first stage, the only instruments available are those of perception through the sensory apparatus that result from direct observation. The symbolic systems for representing the observed patterns are cognitive and are driven by natural language and sketches. One can confidently say that the Bharateeyas, belonging to Bharatavarsha, have contributed immensely towards the development of models with a very scientifically developed Sanskrit language and their very keen observation skills. The main objective of this paper is to present the Vaisesika model enunciated by Rishi Kanada at least 8000 years ago [2].

While Kanada, in his Vaiśeṣika Sutras, considered only six padharthas to be enough to understand or gain knowledge of the fundamental principles in nature, Guatama, in his Nyaya sutras, considered as many as 14 padharthas. Currently, Taraksangraha [3] considers that the six of Kanada and Abhava as enough.

GOD is Omnipresence, Omniscience and Omnipotence

A Comprehension based on the Concept of Fourier Transform

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Abstract

Purpose: In this paper, the mathematical concept of Fourier transform is being used to *draw a parallel* for developing a conceptual comprehension of the Omnipresence, Omniscience, and Omnipotence.

Background:

(i) *Mathematical Background:* The Fourier transform of a Gaussian function in the time and space domain results in a Gaussian function in the Fourier space of temporal and spatial frequencies. The widths of the Gaussian functions in both the domains are inversely related. That is, the smaller the width in the space-time domains the larger the width in their respective Fourier domains. When the width of the Gaussian function with unit area tends to zero with the total area under the curve remaining as unity, we call the resulting function a Delta function. The Fourier transform of such a Delta function would become a constant function in the Fourier domain.

(ii) *Vedantic Background:* Anantam means na antam, that which has no limits or is Infinite. In what sense? Typically, one finds that there are three kinds of limitations one observes:

- Spatial limitation: Every object in the Universe, however big it might be, has a certain limit. The Ocean and the Earth, even though they look limitless, have their limited sizes. All living and non-living things are also limited in size.
- Temporal limitation: Every entity similarly has a beginning and an end. Our body comes into existence at birth and goes out of existence at death and hence has a limited existence in time. Even the Universe is created and dissolved over a very large time frame but yet has this limitation in time.
- Identity limitation: Every object or entity is limited to be of only one type and can not be of another type. A pen can not be a book and vice-versa. So, each object has an identification based on its characteristics and is limited by those characteristics.

Hence, Brahman being Anantam, has no limits in space, time, or identity. That is,

- ब्रह्म (Brahman) is all-pervading or there is no place where ब्रह्म (Brahman) is not, which means Omnipresence. ब्रह्म (Brahman) is present at all times past, present, and future, which means ब्रह्म (Brahman) is Nitya or Eternal and hence is Omniscient.
- ब्रह्म (Brahman) is present in every entity irrespective of its identity. That is, there exists no entity that ब्रह्म (Brahman) is not. This means, ब्रह्म (Brahman) is Non-dual or अद्वैतम् (advaitam).